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CONTINUOUS FIELDS OF PROPERLY INFINITE C*-ALGEBRAS

ETIENNE BLANCHARD

ABSTRACT. Any unital separable continuous $C(X)$ -algebra with properly infinite fibres is properly infinite as soon as the compact Hausdorff space X has finite topological dimension. We show that this is still true when the compact space X has infinite topological dimension.

1. INTRODUCTION

One of the basic C*-algebras studied in the classification programme launched by G. Elliott ([Ell94]) of nuclear C*-algebras through K-theoretical invariants is the Cuntz C*-algebra \mathcal{O}_∞ generated by infinitely many isometries with pairwise orthogonal ranges ([Cun77]). This C*-algebra is pretty rigid in so far as it is a strongly self-absorbing C*-algebra ([TW07]): Any separable unital continuous $C(X)$ -algebra A the fibres of which are isomorphic to the same strongly self-absorbing C*-algebra D is a trivial $C(X)$ -algebra provided the compact Hausdorff base space X has finite topological dimension. (Indeed, the strongly self-absorbing C*-algebra D tensorially absorbs the Jiang-Su algebra \mathcal{Z} ([Win09]). Hence, this C*-algebra D is K_1 -injective ([Rør04]) and the $C(X)$ -algebra A satisfies $A \cong D \otimes C(X)$ ([DW08]).) But I. Hirshberg, M. Rørdam and W. Winter have built a non-trivial unital continuous C*-bundle over the infinite dimensional compact product $\prod_{n=0}^\infty S^2$ such that all its fibres are isomorphic to the strongly self-absorbing UHF algebra of type 2^∞ ([HRW07, Example 4.7]). More recently, M. Dădărlat has constructed in [Dăd09, §3] for all pair (Γ_0, Γ_1) of discrete countable torsion groups a unital separable continuous $C(X)$ -algebra A such that:

- the base space X is the compact Hilbert cube $X = \mathfrak{X}$ of infinite dimension,
- all the fibres A_x ($x \in \mathfrak{X}$) are isomorphic to the strongly self-absorbing Cuntz C*-algebra \mathcal{O}_2 generated by two isometries s_1, s_2 satisfying $1_{\mathcal{O}_2} = s_1 s_1^* + s_2 s_2^*$,
- $K_i(A) \cong C(Y_0, \Gamma_i)$ for $i = 0, 1$, where $Y_0 \subset [0, 1]$ is the canonical Cantor set.

These K -theoretical conditions imply that the $C(X)$ -algebra A is not a trivial one. But these arguments does not anymore work when the strongly self-absorbing algebra D is the Cuntz algebra \mathcal{O}_∞ ([Cun77]), in so far as $K_0(\mathcal{O}_\infty) = \mathbb{Z}$ is a torsion free group.

We still do not know whether all unital continuous $C(X)$ -algebra with fibres \mathcal{O}_∞ are trivial. But we show that any unital separable continuous $C(X)$ -algebra with fibres \mathcal{O}_∞ is at least properly infinite (see [Blan13, §6]).

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2. NOTATIONS

We present in this section the main notations which are used in this article. We denote by $\mathbb{N} = \{0, 1, 2, \dots\}$ the set of positive integers and we denote by $[S]$ the closed linear span of any subset S in a Banach space.

Definition 2.1. ([Dix69], [Kas88], [Blan97]) *Let X be a compact Hausdorff space and let $C(X)$ be the C^* -algebra of continuous function on X .*

- *A unital $C(X)$ -algebra is a unital C^* -algebra A endowed with a unital morphism of C^* -algebra from $C(X)$ to the centre of A .*
- *For all closed subset $F \subset X$ and all element $a \in A$, one denotes by $a|_F$ the image of a in the quotient $A|_F := A/C_0(X \setminus F) \cdot A$. If $x \in X$ is a point in X , one calls fibre at x the quotient $A_x := A|_{\{x\}}$ and one write a_x for $a|_{\{x\}}$.*
- *The $C(X)$ -algebra A is said to be continuous if the upper semicontinuous map $x \in X \mapsto \|a_x\| \in \mathbb{R}_+$ is continuous for all $a \in A$.*

Remarks 2.2. a) ([Cun81], [BRR08]) For all integer $n \geq 2$, the C^* -algebra $\mathcal{T}_n := \mathcal{T}(\mathbb{C}^n)$ is the universal unital C^* -algebra generated by n isometries s_1, \dots, s_n satisfying the relation

$$s_1 s_1^* + \dots + s_n s_n^* \leq 1. \quad (2.1)$$

b) A unital C^* -algebra A is *properly infinite* if and only if one the following equivalent conditions holds ([Cun77], [Rør03, Proposition 2.1]):

- the C^* -algebra A contains two isometries with mutually orthogonal range projections, *i.e.* A unittally contains a copy of \mathcal{T}_2 ,
- the C^* -algebra A contains a unital copy of the simple Cuntz C^* -algebra \mathcal{O}_∞ generated by infinitely many isometries with pairwise orthogonal ranges.

3. GLOBAL PROPER INFINITENESS

Proposition 2.5 of [BRR08] and section 6 of [Blan13] entail the following stable proper infiniteness for continuous $C(X)$ -algebras with properly infinite fibres.

Proposition 3.1. *Let X be a second countable perfect compact Hausdorff space, *i.e.* without any isolated point, and let A be a separable unital continuous $C(X)$ -algebra with properly infinite fibres.*

1) *There exist:*

- (a) *a finite integer $n \geq 1$,*
- (b) *a covering $X = \overset{\circ}{F}_1 \cup \dots \cup \overset{\circ}{F}_n$ by the interiors of closed balls F_1, \dots, F_n ,*
- (c) *unital embeddings of C^* -algebra $\sigma_k : \mathcal{O}_\infty \hookrightarrow A|_{F_k}$ ($1 \leq k \leq n$).*

2) *The tensor product $M_p(\mathbb{C}) \otimes A$ is properly infinite for all large enough integers p .*

Proof. 1) For all point $x \in X$, the semiprojectivity of the C^* -subalgebra $\mathcal{O}_\infty \hookrightarrow A_x$ ([Blac04, Theorem 3.2]) entails that there are a closed neighbourhood $F \subset X$ of the point x and a unital embedding $\mathcal{O}_\infty \otimes C(F) \hookrightarrow A|_F$ of $C(F)$ -algebra. The compactness of the topological space X enables to conclude.

2) Proposition [BRR08, Proposition 2.7] entails that $M_{2^{n-1}}(A)$ is properly infinite and [Rør97, Proposition 2.1] implies that $M_p(A)$ for all integer $p \geq 2^{n-1}$. \square

Remark 3.2. If X is an ordinary second countable compact Hausdorff space and A is a separable unital continuous $C(X)$ -algebra, then $\tilde{X} := X \times [0, 1]$ is a perfect compact space, $\tilde{A} := A \otimes C([0, 1])$ is a unital continuous $C(\tilde{X})$ -algebra and any unital morphism $\mathcal{O}_\infty \rightarrow \tilde{A}$ induces a unital morphism $\mathcal{O}_\infty \rightarrow A$ by composition with the projection map $\tilde{A} \rightarrow A$ coming from the injection $x \in X \mapsto (x, 0) \in \tilde{X}$.

The proper infiniteness of the tensor product $M_p(\mathbb{C}) \otimes A$ does not always imply that the C^* -algebra A is properly infinite ([HR98]). Indeed, there exists a unital C^* -algebra A which is not properly infinite, but such that the tensor product $M_2(\mathbb{C}) \otimes A$ is properly infinite ([Rør03, Proposition 4.5]). We nevertheless have the following corollary.

Corollary 3.3. *Let j_0, j_1 denote the two canonical unital embeddings of the Cuntz extension \mathcal{T}_2 in the full unital free product $\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2$ and let $\tilde{u} \in \mathcal{U}(\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2)$ be a K_1 -trivial unitary satisfying $j_1(s_1) = \tilde{u} \cdot j_0(s_1)$ ([BRR08, Lemma 2.4]).*

Then the following conditions are equivalent:

- (a) *The full unital free product $\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2$ is K_1 -injective.*
- (b) *The unitary \tilde{u} belongs to the connected component $\mathcal{U}_0(\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2)$ of $1_{\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2}$.*
- (c) *Every separable unital continuous $C(X)$ -algebra A with properly infinite fibres is a properly infinite C^* -algebra.*

Proof. (a) \Rightarrow (b) A unital C^* -algebra A is called K_1 -injective if and only if all K_1 -trivial unitaries $v \in \mathcal{U}(A)$ are homotopic to the unit 1_A in $\mathcal{U}(A)$ (see *e.g.* [Roh09]). Thus, (b) is a special case of (a) since $K_1(\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2) = \{1\}$ (see *e.g.* [Blan10, Lemma 4.4]).

(b) \Rightarrow (c) Let A be a separable unital continuous $C(X)$ -algebra with properly infinite fibres. Take a finite covering $X = \overset{o}{F}_1 \cup \dots \cup \overset{o}{F}_n$ such that there exist unital embeddings $\sigma_k : \mathcal{T}_2 \rightarrow A|_{F_k}$ ($1 \leq k \leq n$). Set $G_k := F_1 \cup \dots \cup F_k \subset X$ for all $1 \leq k \leq n$ and let us construct by induction isometries $w_k \in A|_{G_k}$ such that the two projections $w_k w_k^*$ and $1_{|G_k} - w_k w_k^*$ are properly infinite and full in the restriction $A|_{G_k}$:

– If $k = 1$, the isometry $w_1 := \sigma_1(s_1)$ has the requested properties.

– If $k \in \{1, \dots, n-1\}$ and the isometry $w_k \in A|_{G_k}$ is already constructed, then Lemma 2.4 of [BRR08] implies that there exist an homomorphism of unital C^* -algebra $\pi_k : \mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2 \rightarrow A|_{G_k \cap F_{k+1}}$ satisfying:

$$\begin{aligned} - \quad \pi_k(j_0(s_1)) &= w_k|_{G_k \cap F_{k+1}}, \\ - \quad \pi_k(j_1(s_1)) &= \sigma_{k+1}(s_1)|_{G_k \cap F_{k+1}} = \pi_k(\tilde{u}) \cdot w_k|_{G_k \cap F_{k+1}}. \end{aligned} \tag{3.1}$$

If the unitary \tilde{u} belongs to $\mathcal{U}_0(\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2)$, then $\pi_k(\tilde{u})$ is homotopic to $1_{A|_{G_k \cap F_{k+1}}} = \pi_k(1_{\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2})$ in $\mathcal{U}(A|_{G_k \cap F_{k+1}})$, so that $\pi_k(\tilde{u})$ admits a unitary lifting z_{k+1} in $\mathcal{U}_0(A|_{F_{k+1}})$ (see *e.g.* [LLR00, Lemma 2.1.7]). The only isometry $w_{k+1} \in A|_{G_{k+1}}$ satisfying the two constraints:

$$w_{k+1}|_{G_k} = w_k \quad \text{and} \quad w_{k+1}|_{F_{k+1}} = (z_{k+1})^* \cdot \sigma_{k+1}(s_1) \tag{3.2}$$

verifies that the two projections $w_{k+1}w_{k+1}^*$ and $1_{|G_{k+1}} - w_{k+1}w_{k+1}^*$ are properly infinite and full in $A_{|G_{k+1}}$.

The proper infiniteness of the projection $w_n w_n^* \in A_{|G_n} = A$ implies that the unit $1_A = w_n^* w_n = w_n^* \cdot w_n w_n^* \cdot w_n$ is also a properly infinite projection in A , *i.e.* the C^* -algebra A is properly infinite.

(c) \Rightarrow (a) The C^* -algebra $\mathcal{D} := \{f \in C([0, 1], \mathcal{T}_2 *_{\mathbb{C}} \mathcal{T}_2); f(0) \in \mathcal{J}_0(\mathcal{T}_2) \text{ and } f(1) \in \mathcal{J}_1(\mathcal{T}_2)\}$ is a unital continuous $C([0, 1])$ -algebra the fibres of which are all properly infinite. Thus, condition (c) implies that the C^* -algebra \mathcal{D} is properly infinite, a statement which is equivalent to the K_1 -injectivity of $\mathcal{T}_2 *_{\mathbb{C}} \mathcal{T}_2$ ([Blan10, Proposition 4.2]). \square

Proposition 3.4. *Let $a \in \mathcal{T}_2$ be the sum $a := 1 - s_1 s_1^* + s_1$.*

- 1) *The operator a is a unitary in \mathcal{T}_2 .*
- 2) *The unitary $u := \mathcal{J}_1(a) \cdot \mathcal{J}_0(a)^* \in \mathcal{T}_2 *_{\mathbb{C}} \mathcal{T}_2$ belongs to the connected component $\mathcal{U}_0(\mathcal{T}_2 *_{\mathbb{C}} \mathcal{T}_2)$. It also satisfies $\mathcal{J}_1(s_1) \cdot \mathcal{J}_0(s_1^*) = u \cdot \mathcal{J}_0(s_1 s_1^*)$.*
- 3) *The C^* -algebra $\mathcal{T}_2 *_{\mathbb{C}} \mathcal{T}_2$ is K_1 -injective.*

Proof. 1) Let H be the Hilbert space $H := \ell^2(\mathbb{N})$ with canonical orthonormal basis $\{e_k; k \in \mathbb{N}\}$ and identify \mathcal{T}_2 with the C^* -algebra generated by the two isometries s_1, s_2 given by $s_i \cdot e_k := e_{2k+i}$ for $i = 1, 2$ and $k \in \mathbb{N}$.

a is an isometry since $a^* a = ((1 - s_1^* s_1) + s_1)^* ((1 - s_1^* s_1) + s_1) = ((1 - s_1^* s_1) + s_1^* s_1) = 1$. For all vector $\xi \in H$, we also have

$$\begin{aligned} aa^* \xi = 0 &\Leftrightarrow a^* \xi = s_1^* \xi + (1 - s_1 s_1^*)^* \xi = 0 \\ &\Rightarrow (1 - s_1 s_1^*)^* \xi = 0 \text{ and } s_1^* \xi = 0 \\ &\Rightarrow \xi = s_1 s_1^* \xi \text{ and } s_1^* \xi = 0 \\ &\Rightarrow \xi = 0. \end{aligned}$$

Thus, the coisometry a^* is injective and the operator a is a unitary.

2) The unitary a is homotopic to 1 in $\mathcal{U}(\mathcal{T}_2)$ (see *e.g.* [Blan10, Proposition 3.2]). Hence, the product $u = \mathcal{J}_1(a) \cdot \mathcal{J}_0(s_1^*)$ is homotopic to 1 in $\mathcal{U}(\mathcal{T}_2 *_{\mathbb{C}} \mathcal{T}_2)$.

Besides, $u \cdot \mathcal{J}_0(s_1 s_1^*) = \mathcal{J}_1(a) \cdot \mathcal{J}_0(s_1^*) = \mathcal{J}_1(s_1) \cdot \mathcal{J}_0(s_1)^*$.

3) This derives from Corollary 3.3. \square

4. THE PIMSNER-TOEPLITZ ALGEBRA OF A HILBERT $C(X)$ -MODULE

We look in this section at the special case of unital continuous $C(X)$ -algebras with fibres \mathcal{O}_∞ corresponding to the Pimsner-Toeplitz $C(X)$ -algebras of Hilbert $C(X)$ -modules with infinite dimension fibres.

Definition 4.1. ([Pim95]) *Let X be a compact Hausdorff space and E a full Hilbert $C(X)$ -module E , *i.e.* without any zero fibre.*

a) *The full Fock Hilbert $C(X)$ -module $\mathcal{F}(E)$ of E is the direct sum of Hilbert $C(X)$ -module*

$$\mathcal{F}(E) := \bigoplus_{m \in \mathbb{N}} E^{(\otimes_{C(X)} m)}, \quad (4.1)$$

where $E^{(\otimes_{C(\mathfrak{X})})^m} := \begin{cases} C(X) & \text{if } m = 0, \\ E \otimes_{C(X)} \dots \otimes_{C(X)} E \text{ (} m \text{ terms)} & \text{if } m \geq 1. \end{cases}$

b) The Pimsner-Toeplitz $C(X)$ -algebra $\mathcal{T}(E)$ of E is the unital subalgebra of the $C(X)$ -algebra $\mathcal{L}_{C(\mathfrak{X})}(\mathcal{F}(E))$ of adjointable $C(X)$ -linear operator acting on $\mathcal{F}(E)$ generated by the creation operators $\ell(\zeta)$ ($\zeta \in E$), where:

$$\begin{aligned} - \ell(\zeta)(f \cdot \hat{1}_{C(X)}) &:= f \cdot \zeta = \zeta \cdot f & \text{for } f \in C(X) & \text{and} \\ - \ell(\zeta)(\zeta_1 \otimes \dots \otimes \zeta_k) &:= \zeta \otimes \zeta_1 \otimes \dots \otimes \zeta_k & \text{for } \zeta_1, \dots, \zeta_k \in E & \text{if } k \geq 1. \end{aligned} \quad (4.2)$$

c) Let $(C^*(\mathbb{Z}), \Delta)$ be the compact quantum group generated by a unitary \mathbf{u} with spectrum the unit circle and with coproduct $\Delta(\mathbf{u}) = \mathbf{u} \otimes \mathbf{u}$. Then, there is a unique coaction α of the Hopf C^* -algebra $(C^*(\mathbb{Z}), \Delta)$ on the Pimsner-Toeplitz $C(X)$ -algebra $\mathcal{T}(E)$ such that $\alpha(\ell(\zeta)) = \ell(\zeta) \otimes \mathbf{u}$ for all $\zeta \in E$, i.e.

$$\begin{aligned} \alpha : \mathcal{T}(E) &\rightarrow \mathcal{T}(E) \otimes C^*(\mathbb{Z}) &= C(\mathbb{T}, \mathcal{T}(E)) \\ \ell(\zeta) &\mapsto \ell(\zeta) \otimes \mathbf{u} &= (z \mapsto \ell(z\zeta)) \end{aligned} \quad (4.3)$$

The fixed point $C(X)$ -subalgebra $\mathcal{T}(E)^\alpha = \{a \in \mathcal{T}(E); \alpha(a) = a \otimes 1\}$ under this coaction is the closed linear span

$$\mathcal{T}(E)^\alpha = \left[C(X).1 + \sum_{k \geq 1} \ell(E)^k \cdot (\ell(E)^k)^* \right]. \quad (4.4)$$

Besides, the projection $P \in \mathcal{L}(\mathcal{F}(E))$ onto the submodule E induces a quotient morphism of $C(X)$ -algebra $a \in \mathcal{T}(E)^\alpha \mapsto \bar{\mathbf{q}}(a) := P \cdot a \cdot P \in \mathcal{K}(E) + C(X) \cdot 1 \subset \mathcal{L}(E)$.

Proposition 4.2. *Let X be a second countable compact Hausdorff perfect space and let E be a separable Hilbert $C(X)$ -module with infinite dimensional fibres.*

1) *There exist a covering $X = \overset{o}{F}_1 \cup \dots \cup \overset{o}{F}_m$ by the interiors of closed subsets F_1, \dots, F_m and m sections ζ_1, \dots, ζ_m in E such that $\mathcal{T}(E) = C^* \langle \mathcal{T}(E)^\alpha, \ell(\zeta_1), \dots, \ell(\zeta_m) \rangle$ and $\|(\zeta_k)_x\| = 1$ for all $k \in \{1, \dots, m\}$ and $x \in F_k$.*

2) *The Pimsner-Toeplitz $C(X)$ -algebra $\mathcal{T}(E)$ is properly infinite.*

Proof. 1) For all point $x \in X$, there exists a section $\zeta \in E$ satisfying $\|\zeta_x\| = 1$, whence an isomorphism of C^* -algebra $\mathcal{T}(E)_x \cong \mathcal{T}(E_x) = C^* \langle \mathcal{T}(E_x)^\alpha, \ell(\zeta_x) \rangle$. The semiprojectivity of the C^* -algebra $\mathcal{O}_\infty \cong \mathcal{T}(E)_x$ and the compactness of the space X imply that there exist a finite covering $X = \overset{o}{F}_1 \cup \dots \cup \overset{o}{F}_m$ by the interiors of closed subsets F_1, \dots, F_m and m contractions ζ_1, \dots, ζ_m in E such that $\|(\zeta_k)_x\| = 1$ for all index $k \in \{1, \dots, m\}$ and all point $x \in F_k$, so that $\ell(E)_{|F_k} = \ell(E) \cdot \ell(\zeta_k)^* \cdot \ell(\zeta_k)_{|F_k}$ and $\mathcal{T}(E)_{|F_k} = C^* \langle \mathcal{T}(E)^\alpha, \ell(\zeta_k) \rangle_{|F_k}$.

2) The equivalence (a) \Leftrightarrow (c) in Corollary 3.3 implies that $\mathcal{T}(E)$ is properly infinite. \square

Question 4.3. Is there a unitary $v \in \mathcal{U}((\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2)^\alpha)$ such that $j_1(s_1 s_1^*) = v \cdot j_0(s_1 s_1^*) \cdot v^*$?

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